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Diff Calculus (Curvature)

Q.1. For the curve $s = \sqrt{8ay}$, prove that

$$\rho = 4a \sqrt{1 - \frac{y}{2a}}$$

Soln: The equation of the curve is $s = \sqrt{8ay}$ — (1)

Differentiating with respect to y , we get

$$\frac{ds}{dy} = \sqrt{8a} \times \frac{1}{2\sqrt{y}} = \sqrt{\frac{2a}{y}} = \frac{\sqrt{2a}}{s} \times \sqrt{8a} = \frac{4a}{s} \quad [\text{from I}]$$

$$\text{But } \frac{ds}{dy} = \operatorname{cosec} \psi. \therefore \operatorname{cosec} \psi = \frac{4a}{s}$$

Differentiating with respect to s , we get

$$-\operatorname{cosec} \psi \cot \psi \frac{d\psi}{ds} = -\frac{4a}{s^2}$$

$$\text{But } \rho = \frac{ds}{d\psi}$$

$$\therefore \operatorname{cosec} \psi \sqrt{\operatorname{cosec}^2 \psi - 1} \frac{1}{\rho} = \frac{4a}{s^2}$$

$$\text{Hence, } \rho = \frac{s^2}{4a} \times \frac{4a}{s} \sqrt{\frac{16a^2}{s^2} - 1}$$

$$= \sqrt{16a^2 - s^2} = 4a \sqrt{1 - \frac{s^2}{16a^2}}$$

$$= 4a \sqrt{1 - \frac{8ay}{16a^2}} \quad [\text{from I}]$$

$$\rho = 4a \sqrt{1 - \frac{y}{2a}} \quad [\because \rho \text{ be the radius of curvature}]$$

Q.2. Show that, for the curve $s = ae^{\frac{x}{c}}$, $\rho = s \sqrt{(s^2 - c^2)}$, where ρ stands for the radius of curvature at any point of the curve.

Soln: We have, $s = ae^{\frac{x}{c}}$ — (I)

Differentiating (I) with respect to x , we get

$$\frac{ds}{dx} = \frac{a}{e} e^{\frac{x}{a}} = \frac{s}{c} \quad [\text{by I}]$$

But $\frac{ds}{dx} = \sec \psi \therefore \sec \psi = \frac{s}{c} \quad \text{--- (II)}$

Differentiating II with respect to s , we get

$$\sec \psi \cdot \tan \psi \frac{d\psi}{ds} = \frac{1}{c} \quad \text{--- III}$$

But $\rho = \frac{ds}{d\psi}$

$$\therefore \sec \psi \sqrt{\sec^2 \psi - 1} \cdot \frac{1}{\rho} = \frac{1}{c} \quad [\text{from III}]$$

Hence, $\rho = s \sqrt{\frac{s^2}{c^2} - 1}$, [from II]

$$\Rightarrow \rho = s \sqrt{\frac{s^2}{c^2} - 1} \quad \text{proved}$$

Q.3. For the curve, $y^2 = s^2 + c^2$, prove that
 $s = c \tan \psi$ and $\rho = \frac{y^2}{c}$

Soln: We have, $y^2 = s^2 + c^2 \quad \text{--- (I)}$

Differentiating w.r. to x , we get

$$2y \frac{dy}{dx} = 2s \frac{ds}{dx} \therefore y \tan \psi = s \sec \psi$$

i.e. $y \frac{\sin \psi}{\cos \psi} = \frac{s}{\cos \psi} \therefore \sin \psi = \frac{s}{y} \quad \text{--- (II)}$

Now, $\tan \psi = \frac{\sin \psi}{\cos \psi} = \frac{\sin \psi}{\sqrt{1 - \sin^2 \psi}} = \frac{\frac{s}{y}}{\sqrt{1 - \frac{s^2}{y^2}}} = \frac{s}{\sqrt{y^2 - s^2}} = \frac{s}{\sqrt{c^2}} = \frac{s}{c} \quad [\text{by I}]$

Hence, $s = c \tan \psi \quad \text{--- III}$

Diff. w.r. to ψ , we get

$$\begin{aligned} \frac{ds}{d\psi} &= c \sec^2 \psi = c (\sqrt{1 + \tan^2 \psi})^2 = c (1 + \tan^2 \psi) = c \left(1 + \frac{s^2}{c^2}\right) \\ &= c \left(\frac{c^2 + s^2}{c^2}\right) = \frac{y^2}{c} \quad [\text{from I}] \quad \text{[from III]} \end{aligned}$$

Hence, $\rho = \frac{ds}{d\psi} = \frac{y^2}{c} \quad \text{proved}$